Class: VIII Sub: Maths ①

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Chapter 5: Algebraic Expressions and Factorization

EX: 5.1 P.NO: 55

1. Find the following products:

a. $7a \times (-5ab)$

Ans. $(7 \times -5) \times (a \times ab)$

$= -35a^2b$

d. $\frac{4}{7} lm^3n \times \frac{14}{16} m^2n$

Ans. $\left(\frac{4}{7} \times \frac{14}{16}\right) \times (lm^3n \times m^2n)$

$\because x^m \times x^n = x^{m+n}$

$\frac{1}{2} lm^{3+2}n^{1+1} = \frac{1}{2} lm^5n^2$

2 Multiply the following monomials:

a $2xy$; $-5x^2$; $\frac{1}{4}xy^3$

Ans $(2xy) \times (-5x^2) \times (\frac{1}{4}xy^3)$

$(2 \times -5 \times \frac{1}{4}) \times (xy \times x^2 \times xy^3)$

$\frac{-5}{2} x^{1+2+1}y^{1+3} = \frac{-5}{2} x^4y^4$

3 Find the following products

a $(3m-5)(4m+3)$

Ans $(3m-5)\times(4m+3)$

$= 3m(4m+3) - 5(4m+3)$

$= 3m\times4m + 3m\times3 - 5\times4m - 5\times3$

$= 12m^2 + 9m - 20m - 15$

$= 12m^2 - 11m - 15$

d $\left(\frac{3}{4}x - y^2\right)\left(x + \frac{2}{3}y^2\right)$

Ans. $\frac{3}{4}x\left(x+\frac{2}{3}y^2\right) - y^2\left(x+\frac{2}{3}y^2\right)$

$\frac{3}{4}x\times x + \frac{\cancel{3}}{\cancel{4}_2}\times\frac{\cancel{2}}{\cancel{3}}\times(xy^2) - y^2x - \frac{2}{3}y^2\times y^2$

$\frac{3}{4}x^{1+1} + \frac{1}{2}xy^2 - xy^2 - \frac{2}{3}y^{2+2}$

$\frac{3}{4}x^2 - \frac{1}{2}xy^2 - \frac{2}{3}y^4$

4 Multiply the following monomials by trinomials:

a) $(-5b^3)$ by $\left(\frac{4}{5}b^2 + 3b - 1\right)$

Ans $\left(-\cancel{5}b^2\times\frac{4}{\cancel{5}}b^2\right) + \left(-5b^2\times3b\right) + \left(-5b^2\times-1\right)$

$-4b^{2+2} + (-5\times3)\times(b^2\times b) + (-5\times-1)\times b^2$

$-4b^4 + (-15) \times b^{2+1} + 5 \times b^2$

$= -4b^4 - 15b^3 + 5b^2$

d) $(-5ab)$ by $(a^3 + b^3 + 3a^2b + 3ab^2)$

Ans $-5ab \times a^3 + (-5ab \times b^3) + (-5ab \times 3a^2b) +$
$(-5ab \times 3ab^2)$

$= (-5 \times 1) \times (ab \times a^3) + (-5 \times 1) \times (ab \times b^3) + (-5 \times 3)(ab \times a^2b)$
$+ (-5 \times 3) \times (ab \times ab^2)$

$= -5a^{1+3}b - 5ab^{1+3} - 15a^{1+2}b^{1+1} - 15a^{1+1}b^{1+2}$

$= -5a^4b - 5ab^4 - 15a^3b^2 - 15a^2b^3$

5 Use the column method to find the following products:

a $(3a+5)(2a-3)$

Ans

$$\begin{array}{r l}
3a+5 & \\
2a-3 & \\
\hline
6a^2+10a & \text{(multiply by } 2a) \\
-9a-15 & \text{(multiply by } -3) \\
\hline
6a^2+a-15 & \text{(add the terms vertically)}
\end{array}$$

c $\left(\frac{2}{7}l - \frac{1}{4}m\right)\left(\frac{2}{7}l - \frac{1}{4}m\right)$

Ans

$$\frac{2}{7}l - \frac{1}{4}m$$

$$\frac{2}{7}l - \frac{1}{4}m$$

$$\frac{2}{7}l \times \frac{2}{7}l - \frac{2}{7}l \times \frac{1}{4}m$$

$$- \frac{2}{7}l \times \frac{1}{4}m + \frac{1}{4}m \times \frac{1}{4}m$$

$$\left(\frac{2}{7} \times \frac{2}{7}\right) \times (l \times l) - \frac{2}{7}l \times \frac{1}{4}m$$

$$- \frac{2}{7} \times \frac{1}{4}(l \times m) + \frac{1}{4} \times \frac{1}{4}(m \times m)$$

(Add the terms vertically)

$$= \frac{4}{49}l^2 - \frac{2}{28}lm - \frac{2}{28}lm + \frac{1}{16}m^2$$

$$= \frac{4}{49}l^2 - \frac{4}{28}lm + \frac{1}{16}m^2$$

$$= \frac{4}{49}l^2 - \frac{1}{7}lm + \frac{1}{16}m^2$$

6 Multiply the following and verify the result by taking $a = -1$, $b = 2$ and $c = 1$

a $4ab(3a - 2b)$

Ans. $4ab \times 3a - 4ab \times 2b$

$(4 \times 3) \times (ab \times a) - (4 \times 2) \times (ab \times b)$

verify $= 4(-1)(2)(3(-1) - 2(2))$

$= -8(-3-4)$

$= -8(-7)$

$= 56$

(5)

$= 12\ a^{1+1} b - 8\ ab^{1+1}$

$= 12a^2b - 8ab^2$

verify:

$= 12\,(-1)^2(2) - 8(-1)(2)^2$

$= 24 + 32$

$= 56$

d $\left(\frac{1}{2}\alpha - 3\beta\right)^2$

Ans $\left(\frac{1}{2}\alpha - 3\beta\right)\left(\frac{1}{2}\alpha - 3\beta\right)$

$\frac{1}{2}\alpha\left(\frac{1}{2}\alpha - 3\beta\right) - 3\beta\left(\frac{1}{2}\alpha - 3\beta\right)$

$\frac{1}{2}\alpha \times \frac{1}{2}\alpha - \frac{1}{2}\alpha \times 3\beta - 3\beta \times \frac{1}{2}\alpha + 3\beta \times 3\beta$

$\frac{1}{4}\alpha^{1+1} - \left(\frac{1}{2} \times 3\right) \times (\alpha\beta) - 3 \times \frac{1}{2}(\beta\alpha) + (3\times3)(\beta\times\beta)$

$= \frac{1}{4}\alpha^2 - \frac{3}{2}\alpha\beta - \frac{3}{2}\alpha\beta + 9\beta^2$

$= \frac{1}{4}\alpha^2 - \frac{6}{2}\alpha\beta + 9\beta^2$

$= \frac{1}{4}\alpha^2 - 3\alpha\beta + 9\beta^2$

verify: $\frac{1}{4}\alpha^2 - 3\alpha\beta + 9\beta^2 \quad \alpha = -1,\ \beta = 2$

$\frac{1}{4}(-1)^2 - 3(-1)(2) + 9(2)^2$

$+\frac{1}{4} + 6 + 9 \times 4 = +\frac{1}{4} + 6 + 36$

$6 + 36 + \frac{1}{4} = \frac{4 \times 42 + 1}{4} = \frac{168 + 1}{4} = \boxed{\frac{169}{4}}$

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$$\left(\frac{1}{2}\alpha-3\beta\right)^2=\left(\frac{1}{2}(-1)-3(2)\right)^2$$

$$=\left(-\frac{1}{2}-6\right)^2=\left(\frac{-13}{2}\right)^2=\boxed{\frac{169}{4}}$$

8 Find the product of the polynomials $(2x^2-5x+1)$ and $(7x-5x^2+3x+2)$

Ans. $2x^2\times 7x-5x^2\times 2x^2+2x^2+3x+2x^2\times 2$
$-5x(7x-5x^2+3x+2)+1(7x-5x^2+3x+2)$

$(2\times 7)x^3-(5\times 2)x^4+(2\times 3)x^3+(2\times 2)x^2$
$-(5\times 7)(x\times x)-(5\times -5)\times(x\times x^2)-(5\times 3)(x\times x)$
$-(5\times 2)x+7x-5x^2+3x+2$

$14x^3-10x^4+6x^3+4x^2-35x^2+25x^3$
$-15x^2-10x+7x-5x^2+3x+2$

(Add the corresponding co-efficients)

$=(14+6+25)x^3+(-10x^4)+(4-35-15-5)x^2$
$+(\underset{(0)}{-10+7+3})x+2$

$=-10x^4+45x^3-51x^2+2$

EX : 5.2 P.NO : 58

1. Divide the following.

a $-25mn^3$ by $5mn$

Ans. The quotient of the numerical coefficient is $(-25 \div 5) = -5$

Quotient of the variables $= \frac{mn^3}{mn}$

$m^{1-1} n^{3-1} = m^0 n^2 = n^2$

Therefore $-25mn^3 \div 5mn = -5n^2$

b $48p^2q^3r^4$ by $(-8pqr^2)$

Ans The quotient of the numerial coefficient is $48 \div -8 = -6$

quotient of the variables $= \frac{p^2q^3r^4}{pqr^2}$

$= p^{2-1} q^{3-1} r^{4-2} = pq^2r^2$

Therefore $48p^2q^3r^4 \div (-8pqr^{2}) = pq^2r^2$

2 Divide the following

a $12a^3 + 20a^2 - 28a$ by $4a$

Ans $\dfrac{12a^3 + 20a^2 - 28a}{4a} = \dfrac{12a^3}{4a} + \dfrac{20a^2}{4a} - \dfrac{28a}{4a}$

$= 3a^{3-1} + 5a^{2-1} - 7a^{1-1}$ $\boxed{\because a^0 = 1}$

$= 3a^2 + 5a - 7$

c $x^2 + y^2 + 4z^2 + 2xy - 4yz - 4xz$ by $(x+y-2z)$

Ans $x + y - 2z \longrightarrow$ quotient

$x+y-2z \,\big)\, x^2 + y^2 + 4z^2 + 2xy - 4yz - 4xz$

$\cancel{x^2} \quad (-)xy \quad \overset{-}{(+)}2xz$ (sub)

$y^2 + 4z^2 + xy - 4yz - 2xz$ (take other term also)

Subtract $\leftarrow$ $(-)\cancel{y^2} \quad (\mp)\cancel{xy} \quad \overset{-}{(+)}2yz$

take remaining term

$4z^2 \quad -2yz - 2xz$

sub $\leftarrow$ $(-)\cancel{4z^2} \quad \overset{-}{(+)}2yz \quad \overset{-}{(+)}2xz$

$0 \longrightarrow$ Remainder.

step 1: we need x^2, we have x multiply with x means we can get x^2 so, multiply by x.

$(x+y-2z)\,x = x^2+xy-2xz$

write it on the corresponding variables

step 2: Next we need y^2, so we have to multiply y with $(x+y-2z)$

$(x+y-2z)\,y = xy+y^2-2yz$.

step 3: Next we need $4z^2$, so we have to multiply $-2z^2$ with $-2z^2$ we can get $4z^2$

$(x+y-2z)(-2z) = -2xz-2yz+4z$

3 Find the quotients and remainders in each of the following:

$6p^4-5p^3+8p^2-4p+7$ by $(p+1)$

a

$$
\begin{array}{r|l}
 & 6p^3-11p^2+19p-23 \\
\hline
p+1 & 6p^4-5p^3+8p^2-4p+7 \\
 & \cancel{6p^4} \pm 6p^3 \\
\hline
 & -11p^3+8p^2 \\
 & \cancel{-11p^3} \mp 11p^2 \\
\hline
 & 19p^2-4p \\
 & \cancel{19p^2} \pm 19p \\
\hline
 & -23p+7 \\
 & \cancel{-23p} \mp 23 \\
\hline
 & 30
\end{array}
$$

$\therefore p \times 6p^3 = 6p^4$

$\therefore p \times -11p^2 = -11p^3$

$\therefore p \times 19p = 19p^2$

$\therefore p \times -23 = -23p$

Quotient $= 6P^3 - 11P^2 + 19P - 23$

Remainder $= 30$

4 Divide and verify the results:

a $3x^3 - 5x^2 - 11x - 3$ by $(x-3)$

Ans.

$$
\begin{array}{r|l}
 & 3x^2 + 4x + 1 \\
\hline
x-3 & 3x^3 - 5x^2 - 11x - 3 \\
 & \underline{\cancel{3x^3} - 9x^2} \\
 & \quad (-)\ \ (+) \\
 & \quad 4x^2 - 11x \\
 & \quad \underline{\cancel{4x^2} - 12x} \\
 & \quad (-)\ \ (+) \\
 & \qquad x - 3 \\
 & \qquad \underline{\cancel{x} - \cancel{3}} \\
 & \qquad (-)\ \ (+) \\
 & \qquad\quad 0
\end{array}
$$

$\because x \times 3x^2 = 3x^3$

$\because x \times 4x = 4x^2$

$\because 1 \times (x-3) = x-3$

verify:

$(3x^2 + 4x + 1)(x-3) = 3x^3 - 5x^2 - 11x - 3$

LHS: $x(3x^2 + 4x + 1) - 3(3x^2 + 4x + 1)$

$3x^3 + 4x^2 + x - 9x^2 - 12x - 3$

$= 3x^3 - 5x^2 - 11x - 3 = \text{RHS}$.

LHS = RHS.

verified.

5 check by long division method whether x^2-2 is a factor of $2x^4-3x^3-3x^2+6x-2$ or not.

Ans.

$$
\begin{array}{r|l}
 & 2x^2-3x+1 \\
\hline
x^2-2 & 2x^4-3x^3-3x^2+6x-2 \\
 & \cancel{2x^4}+0x^3-4x^2 \quad (-)\;(+) \\
\hline
 & -3x^3+x^2+6x \\
 & (+)\cancel{-3x^3} \quad (-)\cancel{+6x} \\
\hline
 & x^2-2 \\
 & (-)\cancel{x^2} \quad (+)\cancel{-2} \\
\hline
 & 0
\end{array}
$$

$\because x^2 \times 2x^2 = 2x^4$

$\because x^2 \times -3x$

$\because x^2 \times 1 = x^2$

EX: 5.3 P.NO: 61

1 Expand the following using suitable identities

a) $(a+7b)^2$

Ans We know that $(a+b)^2 = a^2+2ab+b^2$

Here $a=a$ & $b=7b$

$(a+7b)^2 = a^2+2(a)(7b)+(7b)^2$

$= a^2+14ab+49b^2$

d) b) $\left(5m+\frac{1}{4}n\right)^2$

$(a+b)^2 = a^2+2ab+b^2$

Here $a=5m$ and $b=\frac{1}{4}n$

$$\left(5m+\frac{1}{4}n\right)^2 = (5m)^2 + 2(5m)\left(\frac{1}{4}n\right) + \left(\frac{1}{4}n\right)^2$$

$$= 5^2m^2 + \frac{2\times5}{4}mn + \frac{1}{16}n^2$$

$$= 25m^2 + \frac{5}{2}mn + \frac{1}{16}n^2$$

2 Use identities to find the products of the following:

a) $\left(6m - \frac{1}{n}\right)\left(6m + \frac{1}{n}\right)$

Ans we know that $(a-b)(a+b) = a^2 - b^2$
Here $a = 6m$ & $b = \frac{1}{n}$

$$\left(6m-\frac{1}{n}\right)\left(6m+\frac{1}{n}\right) = (6m)^2 - \left(\frac{1}{n}\right)^2$$

$$= 36m^2 - \frac{1}{n^2}$$

3 Find the squares of each of the following:

a $b - \frac{1}{4}$

Ans square of $\left(b-\frac{1}{4}\right) = \left(b-\frac{1}{4}\right)^2$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$a = b$ & $b = \frac{1}{4}$

$$= b^2 - 2(b)\frac{1}{4} + \left(\frac{1}{4}\right)^2$$

$$= b^2 - \frac{b}{2} + \frac{1}{16}$$

c $m+5n$

square of $m+5n = (m+5n)^2$

$a = m$ & $b = 5n$

$$(m+5n)^2 = m^2 + 2(m)(5n) + (5n)^2$$

$$= m^2 + 10mn + 25n^2$$

4 Evaluate the following without actual multiplication:

a 96^2

Ans $96 = 100 - 4$

$96^2 = (100-4)^2$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$= 10{,}000 - 2(100)(4) + 16$$

$$= 10{,}000 - 800 + 16$$

$$= 9{,}216$$

c 48×52

Ans $48 = 50 - 2$

$52 = 50 + 2$

$48 \times 52 = (50-2)(50+2)$

$$(a-b)(a+b) = a^2 - b^2$$

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$a = 50$ & $b = 2$

$a^2 - b^2 = 50^2 - 2^2$

$= 2500 - 4$

$= 2{,}496$

5 If $(x+y) = 11$ and $xy = 2$; then find the values of $x^2 + y^2$

Ans $(a+b)^2 = a^2 + 2ab + b^2$

$(x+y)^2 = x^2 + 2xy + y^2$

sub $x+y = 11$, $xy = 2$

$11^2 = x^2 + 2(2) + y^2$

$11^2 = x^2 + 4 + y^2$

$121 - 4 = x^2 + y^2$

$x^2 + y^2 = 117$

6 If $(x-y) = 13$ and $xy = -5$, then find the value of $\frac{1}{3}(x^2 + y^2)$

Ans $(a-b)^2 = a^2 - 2ab + b^2$

sub $x-y = 13$ & $xy = -5$

$(x-y)^2 = x^2 - 2xy + y^2$

$13^2 = x^2 - 2(-5) + y^2$

$169 = x^2 + 10 + y^2$

$169 - 10 = x^2 + y^2 = 159$

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$$x^2+y^2 = 159$$

$$\frac{1}{3}(x^2+y^2) = \frac{159}{3} = 53$$

$$\frac{1}{3}(x^2+y^2) = 53$$

7 If $\left(x-\frac{1}{x}\right) = 15$, then find the value of $\left(x^2+\frac{1}{x^2}\right)$

Ans $\left(x-\frac{1}{x}\right)^2 = x^2 - 2(x)\frac{1}{x} + \left(\frac{1}{x}\right)^2$

$$15^2 = x^2 - 2 + \frac{1}{x^2}$$

$$225 = x^2 + \frac{1}{x^2} - 2$$

$$x^2+\frac{1}{x^2} = 225+2 = 227$$

$$x^2+\frac{1}{x^2} = 227$$

9 If $\left(x-\frac{1}{x}\right) = \sqrt{3}$, then find the value of $\left(x^2+\frac{1}{x^2}\right)$ and $\left(x^4+\frac{1}{x^4}\right)$

Ans $\left(x-\frac{1}{x}\right) = \sqrt{3}$

$$(a-b)^2 = a^2-2ab+b^2$$

$$\left(x-\frac{1}{x}\right)^2 = x^2 - 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$$

$$(\sqrt{3})^2 = x^2 - 2 + \frac{1}{x^2}$$

$$3 = x^2 + \frac{1}{x^2} - 2$$

$$\boxed{x^2 + \frac{1}{x^2} = 5}$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (x^2)^2 + 2(x^2)\left(\frac{1}{x^2}\right) + \left(\frac{1}{x^2}\right)^2$$

$$5^2 = x^4 + 2 + \frac{1}{x^4} \Rightarrow 25 - 2 = x^4 + \frac{1}{x^4}$$

$$\boxed{23 = x^4 + \frac{1}{x^4}}$$

10 Evalute the given algebraic expressions for the values indicated against each of them

a $4p^2 - 20p + 25$ for $p = -2$

Ans $4(-2)^2 - 20(-2) + 25$

$= 4(4) + 40 + 25$

$= 16 + 40 + 25 = 81$

Identity 1 : $(x+y)^2 = x^2 + 2xy + y^2$

Identity 2 : $(x-y)^2 = x^2 - 2xy + y^2$

Identity 3 : $(x+y)(x-y) = x^2 - y^2$

EX : 5.4 P.NO: 64

1. Write down all the possible factors

a) $8x^5y^2$

$8x^5y^2 = 2\times2\times2\times x\times x\times x\times x\times x\times y\times y$

$2\times2\times2\times x\times x\times x\times x\times x\times y\times y$. are factors of $8x^5y^2$

d) $18a^3b^4c^2$

$18a^3b^4c^2 = 2\times3\times3\times a\times a\times a\times b\times b\times b\times b\times c\times c$

2 Find the common factors of the following monomials

a $3pq, 21p^2q$

common factor of the numerical coefficient 3, 21 is 3

common factor of the variable is pq.

so, common factor is 3pq.

d $6ax^2b, 48a^2xb^2, 54axb$

common factor of the numerical is 6.

$$\begin{array}{r|l} 6 & 6, 48, 54 \\ \hline & 1, 8, 9 \end{array}$$

common factor of variable (least value) axb

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So common factor = 6 axb

3 Find the common factors of the following expressions:

a) $5m^4 + 25m^2n^3 - 25m^3n^2$

Ans common factor of the numerical coefficient is 5
common factor of the variable m^4, m^2n^3, m^3n^2
is m^2 (m^2 is common for all)
So common factor = $5m^2$

4 Factorize the following:
a) $2a(4x-1) + 5(4x-1)$
Ans. $2a\underline{(4x-1)} + 5\underline{(4x-1)}$ from the two term take $4x-1$ out side
$(4x-1)(2a+5)$
c) $-8(7x+3y) - 2(7x+3y)$
Ans take $7x+3y$ out side we get
$7x+3y(-8-2) = (7x+3y)(-10)$
$-10(7x+3y)$

5 Factorize using suitable groupings.
a $pqr - pq - r + 1$

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Ans $pqr - pq - r + 1$

from pqr & $-pq$ term we take pq out side. and from $-r$ & 1 term we take -1 out side

$pqr - qp - r + 1$

$pq(r-1) - 1(r-1)$

Then take $(r-1)$ outside

$(r-1)(pq-1)$

e $15pq + 15 + 9q + 25p$

Ans $\underbrace{15pq + 9q}_{\text{we take } 3q \text{ out side}} + \underbrace{15 + 25p}_{\text{we take } 5 \text{ out side}}$.

$3q(5p+3) + 5(5p+3)$

Now take $5p+3$ out side

$(5p+3)(3q+5)$

c $9x^2 + 42x + 49 - 84x$

Ans $9x^2 + (42 - 84)x + 49$

$9x^2 - 42x + 49$

$(3x)^2 - 2 \times 3x \times 7 + (7)^2$

write in $(a+b)^2$ format

$a = 3x \quad b = 7$

$a^2 - 2ab + b^2 = (a-b)^2 = (3x-7)^2$

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6 Factorize the following by using suitable identities

a) $4p^2 - 9q^2$

Ans $(2p)^2 - (3q)^2$ → in $a^2 - b^2$ formate

so, $a^2 - b^2 = (a+b)(a-b)$

$(2p)^2 - (3q)^2$ Here, $a = 2p$ $b = 3q$

so, we get

$(2p)^2 - (3q)^2 = (2p+3q)(2p-3q)$

c $a^4 - 81$

Ans $(a^2)^2 - (9)^2$

Here, $a = a^2$ $b = 9$

$a^2 - b^2 = (a+b)(a-b)$

$(a^2)^2 - 9^2 = (a^2+9)(a^2-9)$

7 Factorize the following by using suitable identities:

a) $x^2 - 6x + 9$

Ans. $(x)^2 - 2(x)(3) + (3)^2$

$a = x$ $b = 3$

$x^2 - 2(x)(3) + 3^2 = (x-3)^2$

c $m^4 + 2m^2n^2 + n^4$

Ans $(m^2)^2 + 2(m^2)(n^2) + (n^2)^2$

$a = m^2$ $b = n^2$

$(m^2)^2 + 2(m^2n^2) + (n^2)^2 = (m^2+n^2)^2$